Unsteady Heat Transfer in Laminar Non-Newtonian Boundary Layer over a Wedge

The transient thermal response of a power-law type non-Newtonian, laminar boundary layer flow over a wedge is investigated. Consideration is given to the case of a step change in surface temperature. Details of the transient heat flux and the temperature field are obtained and have been presented graphically. The range of Prandtl numbers investigated is from 5 to 1000 while the viscosity index was allowed to vary from 0.1 to 5.0.

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SCOPE

It is often necessary to compute the unsteady, forced, convective heat transfer from a body whose surface temperature is changing with time. Although the published literature contains several investigations dealing with the steady state heat transfer in non-Newtonian fluids, analyses for the unsteady heat transfer problem are scarce.

The present work is undertaken in order to investigate the unsteady heat transfer in non-Newtonian fluid boundary layer flow over a wedge. The governing boundary layer equations are formulated and converted into ordinary differential equations by appropriate transformations. The solution then is obtained by first transforming the energy equation in the Laplace transform variable and seeking a series expansion. The series solution is constructed so that it is valid for small as well as large times.

CONCLUSIONS AND SIGNIFICANCE

In this paper, an analysis is presented concerning the unsteady heat transfer in a non-Newtonian, laminar boundary layer flow over a wedge. The thermal boundary layer thickness is seen to increase with time as a result of the step change in surface temperature and ultimately coincide with the steady state distribution as time tends to infinity. The transient heat flux increases with increasing Prandtl number and approaches the steady state value for very large times. The time required for reaching the steady state condition may be calculated from the results presented in this paper.

The step function results presented here serve as a fundamental solution, since by a superposition technique these results may be generalized to apply for arbitrary time variation in the surface temperature.

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INTRODUCTION

Even though a lot of research work has been done in the general considerations of linear, Newtonian relationship between the fluid stress and rate of strain, it has been relatively recently that much research is being done on many aspects of the mechanics of fluids possessing non-linear, non-Newtonian relationships between stress and strain. There exists a need for systematic inquiry of fundamental nature into many aspects of non-Newtonian fluids. Fluids such as molten plastics, pulps, slurries, emulsions, etc., are examples of non-Newtonian fluids and are found to have several applications in the chemical, food, construction, petroleum production and power engineering industries.

Non-Newtonian boundary layer theory is relevant to a wide diversity of engineering activities, among which may be cited the possibility of reducing frictional drag on bearings and on immersed bodies such as ship hulls and submarines. Boundary layer development determines the transport phenomena associated with non-Newtonian flow in the entrance regions of tubes and ducts and flow along banks of tubes in the shell side of heat exchangers. Drag on centrifugal pump blades, free convection processes and the development of the entrance region in non-Newtonian thin film reactors and falling film evaporators are determined by boundary layer characteristics. As has been pointed out by Luikov (1969), the aerodynamics of the modern high speed vehicles should be based upon the rheological concepts since atmospheric air at altitudes above 9 km has distinct viscoelastic properties.

In the 1950s, research efforts in non-Newtonian fluids were concentrated on internal flow problems. A review of the subject was given by Metzner (1965). Acrivos et al. (1960) and Shah et al. (1962) were probably the first to study laminar boundary layer flows of non-Newtonian fluids past external surfaces. Wolf and Szewczyk (1966) analyzed the heat heat transfer to an incompressible, laminar flow of power-law, non-Newtonian fluid from an arbitrary symmetrical cylinder. Lee and Ames (1966) found similarity solutions for forced convective heat transfer to power-law fluids in the case of wedge-type flows.

All the previously mentioned investigations deal with steady state heat transfer in non-Newtonian fluids. It is often necessary to compute the unsteady forced convective heat transfer from a body whose surface temperature is changing with time. Neither analyses nor experiments for this condition have been reported in the literature so far.

The present work aims at investigating the unsteady heat transfer in laminar, power-law type, non-Newtonian boundary layer flows over wedges. The transient response behavior and the details of the transient temperature fields for the case of step change in surface temperature are obtained for various Prandtl numbers and a range of values of n, the viscosity index. The unsteady stagnation point heat transfer in non-Newtonian fluids has been studied by Gorla (1980).

GOVERNING EQUATIONS

Let us consider a steady, incompressible, laminar and two-dimensional boundary layer flow of a power law-type, non-Newtonian fluid over a wedge of included angle $\pi\beta$, placed symmetrically in a uniform main stream. We assume the flow to be steady and the free stream temperature to be constant at $T\infty$. Initially the surface temperature is $T\infty$ and at time $t = 0$, a step change in the surface temperature is applied. The coordinate system along with the flow model has been shown in Figure 1.

Assuming constant fluid properties and negligible viscous dissipation, the governing equations under boundary layer approximations can be written as:

$$\text{Mass: } \frac{\partial u^*}{\partial x^*} + \frac{\partial V^*}{\partial y^*} = 0 \quad (1)$$

$$\text{Momentum: } u^* \frac{\partial u^*}{\partial x^*} + V^* \frac{\partial u^*}{\partial y^*} = U^* \frac{\partial U^*}{\partial x^*} + \frac{1}{\rho} \cdot \frac{\partial \tau xy}{\partial y^*} \quad (2)$$

$$\text{Energy: } \frac{\partial T}{\partial t^*} + u^* \frac{\partial T}{\partial x^*} + V^* \frac{\partial T}{\partial y^*} = \alpha \cdot \frac{\partial^2 T}{\partial y^{*2}} \quad (3)$$

$$\text{For Power Law Fluids, } \tau_{xy} = K\left(\frac{\partial u^*}{\partial y^*}\right)^n \quad (4)$$

Here, we note that $U^*(x^*)$ is the velocity outside the boundary layer and is equal to CX^{*m} where C is a constant and $m = \beta/(2-\beta)$. The boundary conditions for the velocity field are:

$$u^*(x^*,0) = V^*(x^*,0) = 0 \text{ and } u^*(x^*,\infty) = U^* \quad (5)$$

The initial and boundary conditions for the temperature field are:

$$T(x^*,y^*,0) = T\infty, T(x^*,0,t^*) = T\infty + (Tw - T\infty)\cdot 1(t^*) \text{ and } T(x^*,\infty,t^*) = T\infty \quad (6)$$

where $1(t^*) = 0$ for $t^* < 0$
$= 1$ for $t^* \geq 0$

COORDINATE TRANSFORMATION

We now define the dimensionless variables

$$x = \frac{x^*}{L} \qquad y = \frac{y^*}{L}\cdot \mathrm{Re}^{1/n+1}$$

$$t = \frac{t^* U_\infty}{L} \qquad u = \frac{u^*}{U_\infty}$$

$$V = \frac{V^*}{U_\infty}\cdot \mathrm{Re}^{1/n+1} \qquad U = \frac{U^*}{U_\infty}$$

$$\mathrm{Re} = \frac{\rho U_\infty^{2-n} L^n}{K} \qquad Pr = \left[\frac{K}{\rho U_\infty^{2-n} L^n}\right]^{2/n+1} \cdot \left(\frac{\rho C p U_\infty L}{k}\right) \quad (7)$$

Defining a stream function Ψ such that $u = d\Psi/dy$ and $v = -d\Psi/dx$ the continuity is automatically satisfied. We may write

$$u = x^m \cdot f'(\eta)$$

$$V = -\left[\frac{1+n}{1-m+2mn}\right]^{1/(1+n)} \cdot x^{(2mn-n-m)/(n+1)} \cdot G(\eta)$$

$$G(\eta) = \left(\frac{2m - nm - 1}{n+1}\right)\cdot(\eta f' - f) + mf$$

$$\eta = \left[\frac{1-m+2mn}{n+1}\right]^{1/(1+n)} \cdot y \cdot x^{(2m-mn-1)/(n+1)} \quad (8)$$

It may be verified that the momentum equation becomes

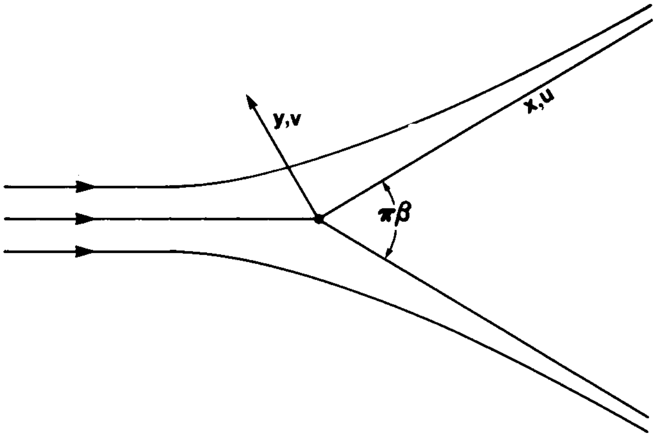

Figure 1. Coordinate system and flow development.

$$\frac{d}{d\eta}(f'')^n + ff'' = \bar{\beta}[(f')^2 - 1] \quad (9)$$

where

$$\bar{\beta} = \frac{m(n+1)}{2mn - m + 1}$$

The transformed boundary conditions are

$$f(0) = f'(0) = 0 \text{ and } f'(\infty) = 1 \quad (10)$$

By means of a numerical solution of Eq. 9, we obtain $f''(0) = a_2$ for various values of m and n. Primes in the above equations designate differentiation with respect to η only.

To transform the energy equation, we define

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\tau = \left[\frac{1-m+2mn}{n+1}\right]^{2/3(1+n)} \cdot t \cdot x^{(6m-2n-4)/3(n+1)}$$

$$\xi = \left[\frac{1-m+2mn}{n+1}\right]^{1/3(1+n)} \cdot y \cdot x^{(3m-n-2)/3(n+1)} \quad (11)$$

The transformed energy equation may be written as

$$\Pr\left\{1 - \frac{2a_2(n+2-3m)}{3(n+1)}\cdot \xi\tau\right\}\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\xi^2} + \Pr\left[\frac{a_2(3m+2n+1)}{6(n+1)}\cdot\xi^2\right]\frac{\partial\theta}{\partial\xi} \quad (12)$$

The initial and boundary conditions are

$$\theta(\xi,0) = 0,\ \theta(0,\tau) = 1(\tau) \text{ and } \theta(\infty,\tau) = 0 \quad (13)$$

SOLUTION

Defining the Laplace transform of θ as

$$\bar{\theta}(\xi,p) = \int_0^\infty e^{-p\tau}\cdot\theta(\xi,\tau)d\tau \quad (14)$$

and applying the Laplace transformation to Eqs. 12 and 13 we have

$$\bar{\theta}'' + \Pr\cdot\mathcal{F}_1(\xi)\bar{\theta}' = \Pr\left[p\bar{\theta} + \mathcal{F}_2(\xi)\cdot\frac{\partial p\bar{\theta}}{\partial p}\right] \quad (15)$$

where

$$\mathcal{F}_1(\xi) = \frac{a_2(3m+2n+1)}{6(n+1)}\cdot\xi^2$$

$$\mathcal{F}_2(\xi) = \frac{2a_2(n+2-3m)}{3(n+1)}\cdot\xi$$

The appropriate boundary conditions are given by

$$\bar{\theta}(0) = \frac{1}{p} \text{ and } \bar{\theta}(\infty) = 0 \quad (16)$$

We now seek a solution of the form

$$\bar{\theta}(\xi,p) = \frac{1}{p}\cdot \exp(R)\cdot \sum_{j=0}^{\infty} u_j(\xi)\cdot [\Pr(p+\lambda)]^{-j/2} \quad (17)$$

where

$$R = -\frac{\Pr\cdot a_2(3m-1)\xi^3}{12(n+1)} - [\Pr(p+\lambda)]^{1/2}\cdot \xi$$

It may be noticed that λ is a function of ξ which is still unknown. We set $u_o(0) = 1$ and $u_j(0) = 0$ for all $j \geq 1$ so that $\bar{\theta}(0) = 1/p$ is identically satisifed. Examining Eq. 17, we see that $\bar{\theta}(\infty) = 0$ is automatically satisifed. Substituting Eq. 17 into 15 and equating coefficients of like powers of $(p + \lambda)$ one obtains a set of ordinary differential equations from which closed form solutions for $u_j(\xi)$ may be obtained. These details are not shown in the interest of conserving space.

It has been found that the first five terms in the series give satisfactory convergence.

Taking the inverse of Eq. 17 we have for the transient temperature field

$$\theta(\xi,\tau) = \exp\left\{-\frac{\Pr a_2(3m-1)}{12(n+1)}\xi^3 - (\Pr\lambda)^{1/2}\xi\right\}\cdot \sum_{j=0}^{\infty} u_j(\xi)\cdot \left(\Pr\lambda\right)^{-j/2}\cdot H_n \quad (18)$$

where

$$H_o = h_1 + h_2$$
$$H_1 = h_1 - h_2 \quad (19)$$
$$H_2 = H_0 - h_3$$

for $j \geq 2$ we have

$$h_1 = \frac{1}{2}\, erfc\left[\left(\frac{\Pr}{2}\right)^{1/2}\left(2\tau\right)^{-1/2}\xi - (\lambda\tau)^{1/2}\right]$$

$$h_2 = \frac{1}{2}\{\exp[2(\Pr\lambda)^{1/2}\xi]\}\cdot erfc\left[\left(\frac{\Pr}{2}\right)^{1/2}\cdot(2\tau)^{-1/2}\xi + (\lambda\tau)^{1/2}\right]$$

$$h_3 = \{\exp[(\Pr\lambda)^{1/2}\xi - \lambda\tau]\}\cdot erfc\left[\left(\frac{\Pr}{2}\right)^{1/2}\cdot(2\tau)^{-1/2}\xi\right]$$

One may notice that H_j varies from 0 to 1 as τ varies from 0 to ∞ and also as ξ varies from 0 to ∞. By letting $\tau \to \infty$ in Eq. 18, we obtain the steady state temperature distribution

$$\theta_s(\xi) = \exp\left\{-\frac{\Pr a_2(3m-1)}{12(n+1)}\xi^3 - (\Pr\lambda)^{1/2}\xi\right\}\cdot \sum_{j=0}^{\infty} u_j(\xi)\cdot(\Pr\lambda)^{-j/2} \quad (20)$$

We now obtain an expression for $\theta'(0,\tau)$ after differentiating Eq. 18 with respect to ξ and substituting $\xi = 0$ as

$$\theta'(0,\tau) = -(\Pr)^{1/2}\cdot(\pi\tau)^{-1/2}\cdot\exp(-\lambda w\tau) - (\Pr\,\lambda w)^{1/2}\cdot erf(\lambda w\tau)^{1/2} + \sum_{j=1}^{\infty} u_j'(0)\cdot(\Pr\lambda w)^{-j/2}\cdot[\Gamma_{\lambda w\tau}(j/2)/\Gamma(j/2)] \quad (21)$$

By letting $\tau \to \infty$, we obtain the steady state value for the wall temperature gradient as

$$\theta'(0,\infty) = \theta_s'(0) = -(\Pr\lambda_w)^{1/2} + \sum_{j=1}^{\infty} u_j'(0)\cdot[\Pr\lambda_w]^{-j/2} \quad (22)$$

λ_w will have to be evaluated from the steady state solution. The steady state problem is described by the following equation

$$\frac{\partial^2\theta s}{\partial\xi^2} + \Pr\left[\frac{a_2(3m+2n+1)}{6(n+1)}\cdot\xi^2\right]\cdot\frac{\partial\theta s}{\partial\xi} = 0 \quad (23)$$

TABLE 1. VALUES OF λ_w FOR VARIOUS VALUES OF n AND Pr

n	λ_w Pr = 5	Pr = 10	Pr = 50	Pr = 100	Pr = 500	Pr = 1000
0.1	1.6947	1.3451	0.7866	0.6243	0.3651	0.2897
0.3	0.9228	0.7324	0.4283	0.3399	0.1988	0.1578
0.5	0.7582	0.6018	0.3519	0.2793	0.1633	0.1296
1.0	0.6515	0.5171	0.3024	0.2400	0.1404	0.1114
3.0	0.6273	0.4979	0.2912	0.2311	0.1351	0.1073
5.0	0.6424	0.5098	0.2982	0.2366	0.1384	0.1098

with boundary conditions

$$\theta_s(0) = 1 \text{ and } \theta_s(\infty) = 0 \quad (24)$$

We now obtain a closed form solution for θ_s as

$$\theta_s(\alpha) = 1 - \frac{\Gamma_\alpha\left(\frac{1}{3}\right)}{\Gamma\left(\frac{1}{3}\right)} \quad (25)$$

where

$$\alpha = a_2\cdot\left[\frac{3m+2n+1}{6(n+1)}\right]\cdot\frac{\Pr}{3}\cdot\xi^3$$

$\Gamma_\alpha(\eta)$ = Incomplete Gamma Function

By differentiating Eq. 25 and then transforming to the ξ-system, we obtain

$$\theta_s'(0) = -\left[a_2\left(\frac{3m+2n+1}{6\{n+1\}}\right)\cdot\frac{\Pr}{3}\right]^{1/3} \Big/ \frac{1}{3}\Gamma\left(\frac{1}{3}\right) \quad (26)$$

By means of Eqs. 20 , 22, 25 and 26, the numerical values of $\lambda(\xi)$ have been obtained through an iterative computer program. A knowledge of the numerical results for λ is essential in obtaining the transient temperature distribution and surface heat flux.

RESULTS AND DISCUSSION

When $\xi = 0$, the series in Eq. 20 becomes identically unity and thus cannot be used to determine λ_w. The value of λ_w was evaluated from Eq. 22 using the values of $\theta_s'(0)$ obtained from Eq. 26. The values of λ_w have been tabulated in Table 1 for a wide range of values of n and Pr. Figure 2 illustrates the growth of the thermal layer with time for $n = 1$. For the sake of brevity, the values of Pr = 10 and $m = 1/3$ (corresponding to a right-angle wedge, $\bar{\beta} = 1/2$)

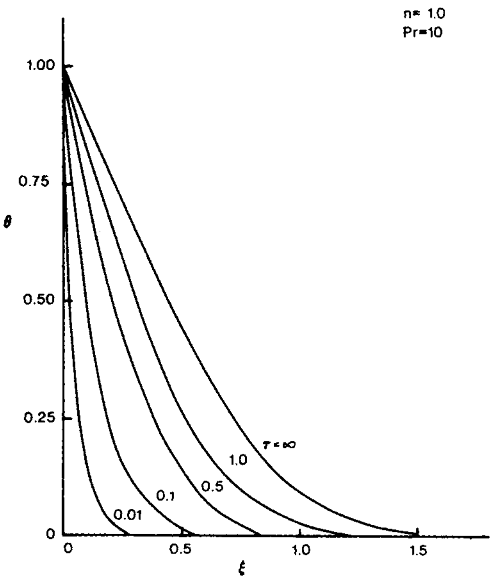

Figure 2. Thermal boundary layer growth for n = 1 and Pr = 10.

have been chosen. The local wall heat flux may be written by Fourier's law as

$$q_w(x^*) = -k\left.\frac{\partial T}{\partial y^*}\right)_{y^*} = 0$$

$$= -k\cdot(T_w - T_\infty)\left[\frac{1-m+2mn}{n+1}\right]^{1/3(1+n)}$$

$$\cdot\, x^{(3m-n-2)/3(n+1)}\cdot\frac{1}{L}\cdot \mathrm{Re}^{1/n+1}\cdot\theta'(0,\tau) \tag{27}$$

The local heat transfer coefficient is given by

$$h(x^*) = \frac{qw}{T_w - T_\infty} = -K\left[\frac{1-m+2mn}{n+1}\right)^{1/3(1+n)}$$

$$\cdot\, x^{(3m-n-2)/3(n+1)}\cdot\frac{1}{L}\cdot \mathrm{Re}^{1/n+1}\cdot\theta'(0,\tau) \tag{28}$$

The local Nusselt number therefore may be written as

$$\mathrm{Nu}x^* = (hx^*/k)$$

$$= -\left(\frac{1-m+2mn}{n+1}\right)^{1/3(1+n)}\cdot x^{3m+2n+1/3(n+1)}$$

$$\cdot\,\mathrm{Re}^{1/n+1}\cdot\theta^1(0,\tau) \tag{29}$$

By letting $\tau \to \infty$ in Eq. 28, one obtains the steady state Nusselt number Nu_s. From Eqs. 26 and 28, we have

$$\mathrm{Nu}_s = \frac{\left[\dfrac{1-m+2mn}{n+1}\right]^{1/3(1+n)}\cdot x^{(3m+2n+1)/3(n+1)}\cdot \mathrm{Re}^{1/n+1}\cdot\left[\dfrac{a_2\cdot Pr}{3}\left(\dfrac{3m+2n+1}{6(n+1)}\right)\right]^{1/3}}{\dfrac{1}{3}\Gamma\left(\dfrac{1}{3}\right)} \tag{30}$$

In the case of a Newtonian fluid ($n = 1$), we obtain for the local Nusselt number under steady state conditions:

$$\mathrm{Nu}_s = 0.4358a_2{}^{1/3}(m+1)^{1/2}\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3} \tag{31}$$

The results for the steady state conditions are in excellent agreement with published results. The ratio of the instantaneous to steady state values of the heat flux are important for practical applications. We may write

$$\frac{q_w}{q_{w,s}} = \frac{\mathrm{Nu}}{\mathrm{Nu}_s} = \frac{\theta'(0,\tau)}{\theta_s'(0)} \tag{32}$$

The ratio of $\mathrm{Nu}/\mathrm{Nu}_s$ can be evaluated by means of Eqs. 21 and 26 for any value of time with Prandtl number and the viscosity index as parameters. The transient heat flux distribution as described by Eq. 32 is illustrated by means of Figure 3 for several values of Pr while n and m were set to be equal to 1 and ⅓ respectively.

From the results obtained, we notice that the thermal boundary layer thickness increases with time following a step change in temperature at the surface and reaches the steady state temperature distribution as $\tau \to \infty$. The thermal boundary layer thickness is seen to be a strong function of the value of the viscosity index. As the value of n increases, the thermal boundary layer thickness is found to increase. The time required to reach the steady state conditions increases with Prandtl number for a given value of n. The time required for approaching steady state conditions may be calculated from the results presented in this paper. As an example, let us consider the boundary layer flow of carbopol 940 solution (0.058% solution) over a flat plate of 1-m length. At a mean temperature of 90°F (32°C), we have $K = 0.0715$ and $n = 0.675$. If the undisturbed free-stream velocity U_∞ is assumed to be 0.1 m/s, the time required to approach steady state conditions (t_s^*) is found to be 85 seconds at $x^* = 0.5$ m. For $x^* = 1$ m, we find that $t_s^* = 123$ seconds.

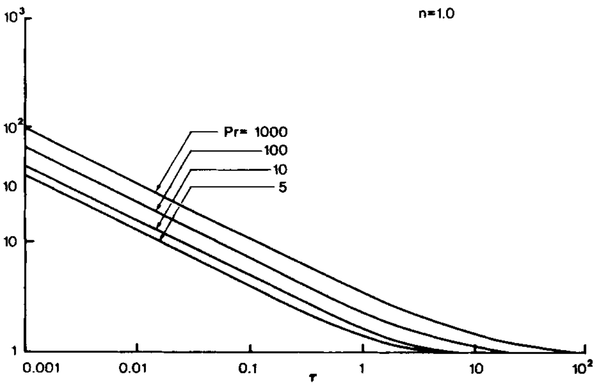

Figure 3. Transient heat flux distribution for $n = 1$ and Pr = 5-1000.

CONCLUDING REMARKS

In this paper, we have presented an analysis investigating the unsteady heat transfer characteristics of a non-Newtonian, laminar boundary layer flow over a wedge. The transient response behavior due to a step change in surface temperature has been considered. In a situation where an arbitrary surface temperature variation is imposed, the heat transfer characteristics can be deduced from the results for the case of a step change in surface temperature. Toward this end, numerical superposition techniques may be used conveniently.

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NOTATION

f	= nondimensional stream function
h	= heat transfer coefficient
K	= viscosity coefficient in Ostwald-de Waele model
k	= thermal conductivity
L	= characteristic length
Nu	= Nusselt number
n	= viscosity index in Ostwald-de Waele model
Pr	= Prandtl number
p	= parameter in Laplace transform
Re	= generalized Reynolds number
T	= temperature
t	= time
U_∞	= free stream velocity
u	= velocity component in streamwise direction
v	= velocity component in transverse direction
x	= coordinate along streamwise direction
y	= coordinate normal to surface
$\Gamma(j)$	= gamma function = $\int_0^\infty \alpha^{j-1}e^{-\alpha}d\alpha$
$\Gamma_x(j)$	= incomplete gamma function = $\int_0^x \alpha^{j-1}e^{-\alpha}d\alpha$
η	= nondimensional coordinate
ξ	= nondimensional coordinate
θ	= nondimensional temperature
ρ	= density

Ψ = stream function
τ = nondimensional time

Subscripts

s = steady state conditions
w = conditions at the wall
∞ = conditions far away from the wall

Superscript

$*$ = physical variables

LITERATURE CITED

Acrivos, A., M. J. Shah, and E. E. Petersen, "Momentum and Heat Transfer in Laminar Boundary Layer Flows of Non-Newtonian Fluids Past External Surfaces," *AIChE J.*, **6**, 312 (1960).

Gorla, R. S. R., "Unsteady Thermal Boundary Layers in Non-Newtonian Fluids," *AIChE Symp. Ser.*, **76**, 264 (1980).

Lee, S. Y. and W. F. Ames, "Similarity Solutions for Non-Newtonians Fluids," *AIChE J.*, **12**, 700 (1966).

Luikov, A. V., "External Convective Mass Transfer in Non-Newtonian Fluid," *Int. J. Heat Mass Transfer*, **12**, 377 (1969).

Metzner, A.B., "Heat Transfer in Non-Newtonian Fluids," *Advances in Heat Transfer*, **2**, 357 (1965).

Shah, M. J., E. E. Petersen, and A. Acrivos, "Heat Transfer from a Cylinder to a Power Law Non-Newtonian Fluid," *AIChE J.*, **8**, 542 (1962).

Wolf, C. J. and A. A. Szewczyk, "Laminar Heat Transfer to Power Law Model Non-Newtonian Fluids from Arbitrary Cylinders," Proceedings of Third International Heat Transfer Conference, **1**, 388 (1966).

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Control System Synthesis Strategies

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This paper is concerned with an important aspect of process control design—synthesis of the control structure. Synthesis of control structures has long been practiced by experienced control engineers, who relied on intuition, insight and judgment to pick a feasible solution from the vast number of alternatives that were possible. This paper describes a systematic procedure to generate these alternatives based on the cause-and-effect representation of the process. The final product is a set of control schemes from which the final system may be selected or evolved. The work is significant in that it is the first attempt to apply non-numerical problem-solving techniques to the problem of synthesizing process control structures. As such, it gives a new way of studying and teaching chemical process control.

SCOPE

Which variables should be measured, which inputs should be manipulated, and what links should be made between these two sets? This is the essence of the synthesis of control structures in the chemical process industries. This problem is routinely solved by experienced engineers who have the ability to simultaneously consider:

1. **The economic, safety and reliability goals of a given process**
2. **The steady-state and dynamic behavior of the complete process and of the pieces of equipment within it**
3. **The interaction which might occur between control structures**
4. **The failure modes of the components within the process including the process operator**
5. **Possible changes in the process to improve control**

These engineers have evolved logical procedures for proceeding from loosely defined flowsheets and goals to well-defined piping and instrument diagrams (P&ID's). Most of these procedures do not involve the use of detailed dynamic models of the process. How do they do it? This paper is our initial attempt to capture at least part of the logic involved in transforming steady-state flow diagrams into P&ID's.

Our approach is based on three main ideas:

1. **The models used in the synthesis of control systems must be simple.**
2. **The propagation of control constraints through the process flow diagram will generate the candidate control structures.**
3. **Evaluation of alternate control structures will depend primarily on: (a) the feasibility and simplicity of measuring and manipulating candidate control structure variables; and (b) the steady-state interactions which occur between control substructures within the process. Detailed dynamic performance needs to be considered only when the alternate control structures have strong dynamic interactions or the dynamic achievements of the control objective is questionable.**

Hence, this approach has a strong component of steady-state control (Buckley, 1964; Shinskey, 1967), integrated with dynamic considerations when it is indicated. These general concepts are not new with us. Control engineers have used these ideas for decades. What we have done is to state the problem and solution method in the language of non-numerical problem-solving (Newell, Shaw and Simon, 1959; Nilsson, 1971). In this language the problem becomes one of using specific rules to generate a search tree of alternate control structures. The alternate structures are evaluated using heuristics derived from both theoretical and practical considerations. The final product is a set of control schemes (say 5 to 10) from which the final system may be selected or evolved.

To test these ideas, computer programs are being developed which, while interacting with the control engineer, will generate and evaluate alternate control systems. With these ideas and computer programs, we hope to clarify the control synthesis problem to generate other research projects.

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